1978/AB5)

Given $x^2 - xy + y^2 = 9$

a) write the general expression for the slope of the curve.

$$2x - \left(y + x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$$

$$-x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x + y$$

$$\frac{dy}{dx} = \frac{-2x+y}{-x+2y}$$

b) Find the coordinates of the points on the curve where the tangents are vertical.

tangents are vertical when

$-x + 2y = 0$ [and $-2x+y$ is not zero]

$x = 2y$ (but NOT $(0,0)$)

$$x^2 - xy + y^2 = 9$$

$$(2y)^2 - (2y)(y) + y^2 = 9$$

$$4y^2 - 2y^2 + y^2 = 9$$

$$3y^2 = 9$$

$$y = \pm\sqrt{3}$$

$x = \pm 2\sqrt{3}$

$(2\sqrt{3}, \sqrt{3})$

$(-2\sqrt{3}, -\sqrt{3})$

c) At the point $(0,3)$ find the rate of change in the slope of the curve wrt x

$$\frac{dy}{dx} = \frac{-2x+y}{-x+2y}$$

$$\frac{d^2y}{dx^2} = \frac{\left(-2+\frac{dy}{dx}\right)(-x+2y) - (-2x+y)\left(-1+2\frac{dy}{dx}\right)}{(-x+2y)^2}$$

at $(0,3)$

$$\frac{dy}{dx} = \frac{-2x+y}{-x+2y}\Bigg| = \frac{3}{6} = \frac{1}{2}$$

6.4/8

$$\left(\frac{1}{1}\right) = \overset{1}{1} - \frac{\overset{2}{1}}{2} + \frac{\overset{3}{1}}{3} - \frac{\overset{4}{1}}{4} + \frac{\overset{5}{1}}{5}$$

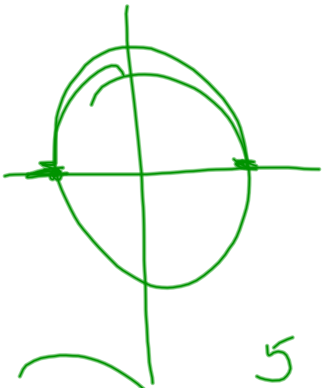
$$\sum_{K=1}^{5} (-1)^{(K+1)} \frac{1}{K} \quad \text{OR} \quad \sum_{K=1}^{5} (-1)^{K} \left(\frac{-1}{K}\right)$$

6.4/1a)

$$\sum_{k=1}^{3} k^3 = 1^3+2^3+3^3$$
$$= 1+8+27 = \underline{36}$$

1f)

$$\sum_{n=1}^{6} \sin(n\pi) = \sin(\pi)+\sin(2\pi)$$
$$+\sin(3\pi)+\sin(4\pi)$$
$$+\sin(5\pi)+\sin(6\pi)$$
$$=0+0+0+0+0+0$$
$$=0$$



1d)

$$\sum_{n=0}^{5} 1 = 1+1+1+1+1+1 = 6$$

6.4/2a)

$$\sum_{K=1}^{4} K\sin\left(\frac{K\pi}{2}\right)$$

$$= \sin\left(\frac{\pi}{2}\right) + \cancel{2\left(\sin\frac{2\pi}{2}\right)} + 3\sin\left(\frac{3\pi}{2}\right) + \cancel{4\sin\left(\frac{4\pi}{2}\right)}$$

$$= 1 + 3(-1) = -2$$

2c)

$$\sum_{i=7 \; (-6)}^{20 \; (-6)} \pi^2 = \pi^2 + \pi^2 + \pi^2 + \cdots\cdots + \pi^2$$

$$= 14\pi^2$$

3.6

20)

$$\frac{xy^3}{1+\sec y} = 1+y^4$$

$$\frac{\left(y^3+x\left(3y^2\frac{dy}{dx}\right)\right)(1+\sec y)-(xy^3)\left(\sec y\tan y\frac{dy}{dx}\right)}{(1+\sec y)^2} = 4y^3\frac{dy}{dx}$$

$$\left(y^3(1+\sec y)\right)+3xy^2(1+\sec y)\frac{dy}{dx}-xy^3\sec y\tan y\frac{dy}{dx} = 4y^3(1+\sec y)^2\frac{dy}{dx}$$

$$\left[-4y^3(1+\sec y)^2+3xy^2(1+\sec y)-xy^3\sec y\tan y\right]\frac{dy}{dx} = -y^3(1+\sec y)$$

$$\frac{dy}{dx} = \frac{-y^3(1+\sec y)}{\left[-4y^3(1+\sec y)^2+3xy^2(1+\sec y)-xy^3\sec y\tan y\right]}$$